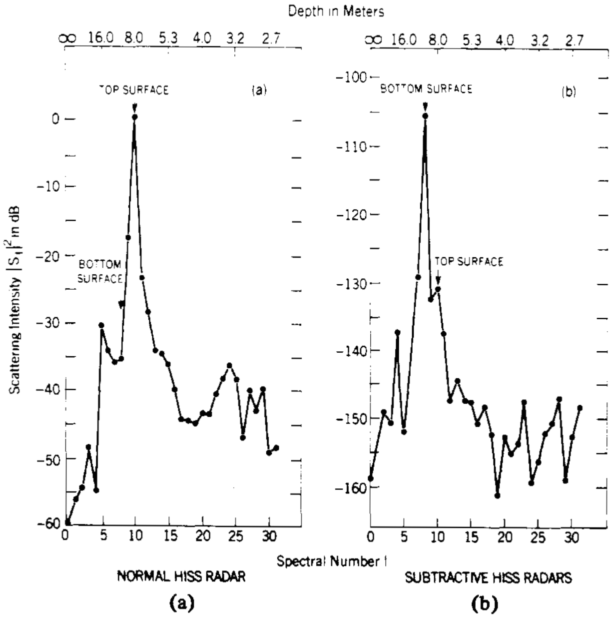

Fig. 3. Comparison of performances of normal and subtractive HISS radars.

from that of Fig. 3(a) by 100 dB. This is equivalent to the expansion of the dynamic range of the radar by as much as 100 dB. The physically realizable value, however, would be much smaller than this.

The achievable sensitivity of the radar will be determined not only by the receiver noise but also by the degrees of error involved in determining the altitude and tilt of the antenna array, and the interpretation of the electrical properties of the surface of the ice.

This proposed method of expanding the dynamic range of a radar is quite unique, and further efforts are being made to implement such a radar system.

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A New Single-Sideband Digital Mixer

JUH TZENG LUE

***Abstract*—A single-sideband digital mixer using logic exclusive-OR gates, *J-K* flip-flops, and one-shot multivibrators, but not containing any reactive filters, accepts square inputs at two arbitrary frequencies f_1 and f_2 and generates a pair of phase-quadrature waves with fundamental frequency at either $f_1 + f_2$ or $f_1 - f_2$.**

A single-sideband frequency converter using exclusive-OR logic gates taking square-wave inputs with output frequencies divided by two and with three-level waveforms has been reported recently [1]. Here, we design a refined sideband digital mixer by using the logic exclusive-OR gates, *J-K* flip-flops, and one-shot multivibrators without applying any reactive filters, which can accept square-wave inputs at two arbitrary frequencies f_1 and f_2 and generate one pair of phase-quadrature output waves with fundamental spectral frequency at either $f_1 + f_2$ or $f_1 - f_2$. The output waveform has two levels corresponding to logical 0 and 1. In this configuration, the undesired higher order components $mf_1 \pm nf_2$, with m and n not equal to 1, have been suppressed relative to the desired upper $f_1 + f_2$ and lower $f_1 - f_2$ sidebands by at least 10 dB according to the calculation of the Fourier series of the signals and the measurement of a sweeping spectrum analyzer. The output signals generated by this method have quadrature phase splittings which are suitable for driving a frequency-to-voltage converter or other phase comparators.

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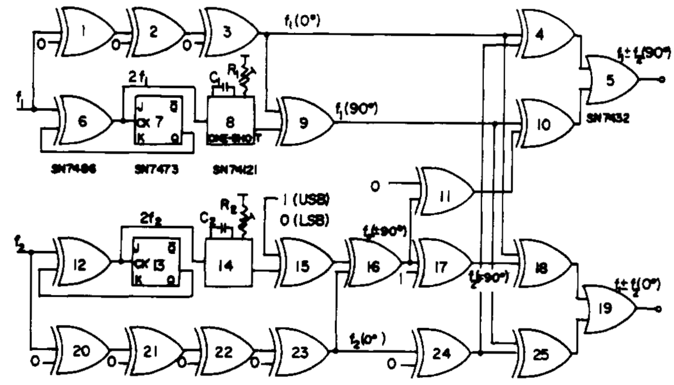

Fig. 1. Circuit detail of the single-sideband digital mixer.

The drawing of the digital mixer is shown in Fig. 1. The operating principal can be summarized as follows. The input frequency is first doubled by application of an exclusive-OR and a flip-flop circuits. This can be obtained by feeding two-phase lagged inputs to exclusive-OR gate 9 or 16. The phase lag is determined by the propagation delay time of the flip-flop 7 and the exclusive-OR gate 6. One-shot multivibrators 8 and 14 are used to restore the symmetry of the waveform, which is very critical to the exact quadrature-phase splitting. The values of the resistors R_1, R_2 and capacitors C_1, C_2, which determine the symmetry of the output waveforms of the monostable multivibrators, are dependent on the input frequencies f_1 and f_2. Integrated circuit 15, gives inverting and noninverting output signals with one of its input gates at "1" and "0" respectively, which determines whether the mixed frequency will be the upper sideband or the lower sideband. The gates with one input tied to zero are used for compensating the propagation delay time of the signal with respect to that of the other channel. An exclusive-OR gate when fed by two in-phase or out-of-phase signals with frequency f and $2f$ will give an output at frequency f with a phase shift of 90°.

A symmetric square-wave signal with period T and time delay t_0 can be represented by a sum of step functions

$$P_T(t) = \sum_{n=-\infty}^{\infty} \left[u(t - t_0 - nT) - u\left(t - t_0 - \frac{2n+1}{2} T\right)\right] \quad (1)$$

where $u(t) = 0$ or 1 with $t < 0$ or $t > 0$ respectively. The Fourier series of the above equation is found readily to be

$$E_f(t) = \sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} \sin\left[2\pi(2k+1)f(t - t_0)\right] \quad (2)$$

where the fundamental frequency f is equal to $1/T$, and k is a positive integer. The output of the exclusive-OR gate with inputs at f and $2f$ is

$$E_f \oplus F_{2f} = \sum_{k'' k'''} \frac{-8}{(k'' + k''' + 1)(k'' - k''')\pi^2} \cdot \left[\cos 2\pi(2k' + 1)ft + \cos 2\pi(2k''' - 1)ft\right] \quad (3)$$

where $k'' = k + 2k' + 1$, $K''' = k - 2k' - 1$. Equation (3) is the Fourier series of a frequency f signal with a phase lag of 90° with respect to $E_f(t)$.

To find the addition of two exclusive-OR gates with quadrature-phase inputs, we first derive the Fourier series of the 90° phase lag signal. This is given by

$$E_f\left(t-\frac{T}{4}\right)=\sum_{k\geq 0}^{\infty}\frac{(-2)(-1)^k}{(2k+1)\pi}\cos\left[2\pi(2k+1)ft\right]. \tag{4}$$

The sum of the two outputs of IC_4 and IC_{10} can be obtained by manipulating the exclusive-OR functions of the quadrature phase inputs given by (2) and (4), and is found to be

$$E_{f_1}(t)\oplus E_{f_2}(t)+E_{f_1}\left(t-\frac{T_1}{4}\right)\oplus E_{f_2}\left(t-\frac{T_2}{4}\right)$$
$$=\sum_{kk'}\frac{-4}{(2k+1)(2k'+1)\pi^2}\cos 2\pi[(2k+1)f_1+(2k'+1)f_2]t. \tag{5}$$

This is the upper sideband frequency. The amplitude of the component at frequency nf_1+mf_2 is suppressed relative to the desired output f_1+f_2 by $20\log(nm)$ db.

This system is used by us in voltage readout of a crystal controlled thin-film thickness monitor, where f_1 is the frequency of the local oscillator and f_2 is subjected to change due to the deposition on the crystal surface. The frequencies f_1 and f_2 are limited by the propagation delay time of the integrated circuits which we apply. The standard SN54/74 integrated circuits are capable of operation with clock input signals as high as 20 MHz. We expect this simple digital mixer can be applied to many applications in communication systems and scientific instruments.

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A Method for Computing Vertical-Plane Coverage Diagrams for Frequency Agile Pulse Radar Systems

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Abstract—**A method for obtaining computer plots of vertical-plane coverage diagrams (i.e., range-height-angle charts) for frequency agile pulse radar systems is presented. The approach is based on well known monochromatic radar detection calculations which are here extended to cover the frequency agile case.**

In a comprehensive report on maximum radar range computations, Blake [1] provides all the necessary tools for obtaining coverage plots (i.e., range-height-angle charts) of monochromatic pulse radar systems. This paper provides a method for extending the range computations applicable to the monochromatic case to pulse radar systems featuring frequency agility. The number of different frequencies employed as well as the number of consecutive pulses to be transmitted at the same frequency are both arbitrary. The detection scheme assumed in the following analysis utilizes a square-law detector followed by a linear integrator as shown in Fig. 1. The symbols D_0, X_0, and Y_0 denote the SNR (i.e., signal-to-noise power ratio) at the designated points of the detector-integrator complex.

Based on approximate closed-form expressions derived by Barton [2] and later simplified by Cann [3], one may relate the input and output SNR's using the equation

$$X_0=\frac{2D_0^2}{D_0+2.3}. \tag{1}$$

The integrated output SNR, Y_0, may now be written in the form

$$Y_0=\sum_{j=1}^{N}X_{0j}=\sum_{j=1}^{N}\left\{\frac{2D_{0j}^2}{D_{0j}+2.3}\right\} \tag{2}$$

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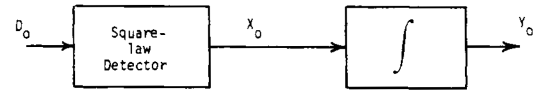

Fig. 1. Square-law detector followed by a linear integrator.

where N denotes the number of integrated pulses, and D_{0j} represents the SNR at the detector input due to the jth pulse illuminating a target located at $R_{\max}$; it is assumed that $R_{\max}$ represents the maximum detection range associated with the total energy delivered on the target by the N transmitted pulses.

Since our objective is to relate the integrated SNR (i.e., Y_0) to a maximum detection range $R_{\max}$ using the classical radar equation, an average SNR is defined at the detector output in the form

$$\hat{X}_0=\frac{1}{N}\sum_{j=1}^{N}X_{0j}=\frac{1}{N}\sum_{j=1}^{N}\frac{2D_{0j}^2}{D_{0j}+2.3}. \tag{3}$$

The SNR given by (3) may be thought of as the SNR necessary to achieve a specified probability of detection P_d and false alarm P_{fa} for a specified number of pulses and a postulated type of target; normally, it is the corresponding SNR, $\hat{D}_0$, at the input of the square-law detector rather than $\hat{X}_0$ that one associates with P_d and P_{fa}. Given, however, $\hat{D}_0$, the corresponding value of $\hat{X}_0$ is obtained using (1). For example, given N, P_d and P_{fa}, and a target described by Swerling's case-2, one may utilize published results [1] to determine $\hat{D}_0$ and subsequently $\hat{X}_0$.

Once $\hat{X}_0$ is obtained, the problem becomes one of relating $\hat{X}_0$ to a corresponding maximum detection range $R_{\max}$ in a way which takes advantage of the computational aids developed by Blake [1], [4]. This is accomplished by writing the radar equation in the form

$$D_{0j}=\frac{P_tA^2\sigma F_j^4}{4\pi\lambda_j^2P_{\text{noise}}\cdot R_{\max}^4}=\left[\frac{P_tA^2\sigma}{4\pi\lambda_j^2\hat{D}_0P_{\text{noise}}}\right]F_j^4\cdot\frac{\hat{D}_0}{R_{\max}^4} \tag{4}$$

where D_{0j} represents the SNR at the detector input due to the jth pulse return from a target positioned at $R_{\max}$; $\hat{D}_0$ is the (averaged) minimum SNR based on N pulses; F_j denotes the pattern-propagation factor, and the remaining symbols in (4) represent well known radar parameters [5]. The bracketed portion of (4) represents the free-space range R_{jfs}^4 corresponding to N pulses, but as if they all had been transmitted at frequency f_j (i.e., wavelength λ_j) and each resulted in a SNR, $\hat{D}_0$, at the detector input. In view of the above interpretation, one may rewrite (4) in the form,

$$D_{0j}=\hat{D}_0\left(\frac{R_{xj}}{R_{\max}}\right)^4 \tag{5}$$

where $R_{xj}=R_{jfs}F_j$, and $j=1,2,\ldots,N$.

Since

$$R_{jfs}^4=\frac{P_tA^2\sigma}{4\pi\lambda_j^2\hat{D}_0P_{\text{noise}}} \tag{6}$$

varying only the frequency results in a relationship of the form

$$R_{jfs}^4=R_{ifs}^4\left(\frac{\lambda_i}{\lambda_j}\right)^2=R_{ifs}^4\left(\frac{f_j}{f_i}\right)^2 \tag{7}$$

which provides the free-space range at frequency f_j in terms of the free-space range at another frequency f_i while all other parameters in (6) remain fixed.

Substituting next (3) and (5) into (1), that is, into expression

$$\hat{X}_0=\frac{2\hat{D}_0}{1+2.3\left(\frac{1}{\hat{D}_0}\right)} \tag{8}$$

one obtains

$$\frac{1}{1+\alpha}=\frac{1}{N}\sum_{j=1}^{N}\left[\frac{(R_{xj}/R_{\max})^4}{1+\alpha(R_{\max}/R_{xj})^4}\right] \tag{9}$$

where $\alpha=(2.3/\hat{D}_0)$.